DAJ-003-1014008 Seat No. ______

B. Sc. (Sem. IV) (CBCS) (W.E.F. 2016) Examination

April / May - 2022

Math : 04(A)

(Linear Algebra & Differential Geometry Theory)

Faculty Code : 003

Subject Code : 1014008

Time : **2.30** Hours] [Total Marks : **70**

Instructions : (1) Attempt any five questions out of ten.

(2) Right hand side digits in square brackets indicates the marks.

1 (A) Answer the following questions in short. **4**

(i) If V is a vector space, then for all $\overline{x}, \overline{y} \in V$; $\alpha, \beta \in \mathbb{R}$ which of the following is not true.

(a) $\overline{x}+\overline{y}=\overline{y}+\overline{x}$ (b) $\overline{x}.\overline{y}=\overline{y}.\overline{x}$

(c) $(\alpha-\beta)\overline{x}=\alpha\overline{x}-\beta\overline{x}$ (d) None of these

(ii) For any non-empty subset A of a vector space V.

(a) Sp $A \subseteq A$ (b) Sp $A = A$

(c) A $\subseteq$ Sp A (d) SP $A = V$

(iii) One vector of the vector set $\{u_1, u_2, ..., u_n\}$ is θ, then the vector set is ________.

(iv) In usual notation define: "$W_1 \oplus W_2$".

(B) Attempt any one out of two: **2**

(i) Prove that sum of two sub space of a vector space is also sub space.

(ii) Prove that the set $\{(1, 2, 1), (-1, 1, 0), (5, -1, 2)\}$ is linearly independent in $\mathbb{R}^3$.

(C) Attempt any one out of two: **3**

(i) Check whether $W = \{(x, y, z) : 2x + 5y - z = 0\}$ is a subspace of $\mathbb{R}^3$ or not.

(ii) In a vector space, prove that non empty subset of a linearly independent set is also linearly independent.

(D) Attempt any one out of two: **5**

(i) State and prove the necessary and sufficient condition for any non empty subset of a vector space to be sub space and hence deduce that span of a non-empty subset of a vector space is a subspace.

(ii) For a non empty set U, show that the power set of U, $P(U)$, becomes a vector space over the field $\mathbb{Z}_2 = \{0, 1\}$ with respect to the following definition of vector addition and scalar multiplication. For $A, B \in P(U)$, $A + B = A \Delta B$; $0A = \phi$; $1A = A$.

2 (A) Answer the following questions in short. **4**

(i) For a basis $A = \{a_1, a_2, \ldots\ldots a_n\}$ of vector space V, if S_1 : "A is linearly independent" and S_2 : "SP $(A) = V$", then

(a) S_1 and S_2 are false

(b) S_1 is true but S_2 is false

(c) S_1 and S_2 are true

(d) S_2 is true but S_1 is false

(ii) If U is sub space of a vector space V, then

(a) dim U = dim V (b) dim $U \leq$ dim V

(c) dim $V \leq$ dim U (d) None of these

(iii) What is the dimension of $M_2(\mathbb{R})$?

(iv) Define : "*Dimension* of a vector space."

(B) Attempt any one out of two : **2**

(i) In a vector space, prove that two basis have the same number of elements.

(ii) Extend the set {(1, 1, 1), (2, 0, 0)} to the basis of $\mathbb{R}^3$.

(C) Attempt any one out of two : **3**

(i) In a vector space, show that any linearly independent set can be extended to the basis.

(ii) Show that $B = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ is a basis of $\mathbb{R}^3$ and write (1, 1, –1) as a linear combination of elements of B.

(D) Attempt any one out of two : **5**

(i) If W_1 and W_2 are two sub spaces of a vector space V, then prove that dim $(W_1 + W_2)$ = dim (W_1) + dim (W_2) – dim $(W_1 \cap W_2)$.

(ii) Prove that $W_1 = \{(x - y, y + z, y, z) : x, y, z \in \mathbb{R})\}$ and $W_2 = \{(x, x + y, x + y + z, y - z : x, y, z \in \mathbb{R})\}$ are sub space of $\mathbb{R}^4$ and hence find dim (W_1), dim (W_2), dim $(W_1 \cap W_2)$ and dim $(W_1 + W_2)$.

3 (A) Answer the following questions in short. **4**

(i) Any linear transformation from $\mathbb{R}^2$ to itself is onto (True/False)

(ii) If rank of linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ is 2 then dimension of *ker* (T) is _______.

(iii) A linear transformation T is one-one if and only if nullity of T is _______.

(iv) A map $T: P_n(\mathbb{R}) \to P_{n+1}(\mathbb{R})$ defined as $T(p(x)) = xp(x), \forall p(x) \in P_n(\mathbb{R})$ is not linear transformation. (True/False)

(B) Attempt any one out of two : **2**

(i) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be defined as $T(x, y) = (x, x + y, y)$, then find rank of T.

(ii) Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be defined as

$T(x, y, z) = (x - y + z, x + y - z)$.

Then find nullity of T.

(C) Attempt any one out of two : **3**

(i) Find T (2, 5, 7). Given that a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(e_1) = (1, 1)$, $T(e_1 + e_2) = (1, 0)$, $T(e_1 + e_2 + e_3) = (1, -1)$, where $\{e_1, e_2, e_3\}$ standard basis of $\mathbb{R}^3$.

(ii) Let $T: V \to V$ be any linear transformation such that $T^2 - T + I = 0$, then prove that T is non singular.

(D) Attempt any one out of two : **5**

(i) State and prove rank-nullity theorem.

(ii) Let $\{v_1, v_2, ... v_n\}$ be a basis of vector space V and let w_i $1 \leq i \leq n$ be any set of (not necessarily distinct) vectors in vector space W. Then show that there is an unique linear transformation $T : V \rightarrow W$ such that $T(v_i) = w_i$.

4 (A) Answer the following questions in short. **4**

(i) Every invertible matrix is diagonalizable. (True/False).

(ii) Write sufficient condition in terms of eigen value for a linear transformation $T : V \rightarrow V$ with $dim(V) = n$ to be diagonalizable.

(iii) Define : 'eigen basis'

(iv) Define : 'dual of vector space'

(B) Attempt any one out of two : **2**

(i) Find eigen values of $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T\ (x,\ y,\ z) = (x,\ 2y,\ 3z)$.

(ii) Define : "adjoint of a linear transformation"

(C) Attempt any one out of two : **3**

(i) Let $T : P_3(\mathbb{R}) \rightarrow P_2(R)$ defined as $T(p(x)) = p'(x)$. Find matrix $[T : B_1, B_2]$ with respect to standard basis $B_1 = \{1, x, x^2, x^3\}$ and $B_2 = \{1, x, x^2\}$.

(ii) Let $T : V \rightarrow V$ be a linear transformation and let B be a basis of V. Then show that λ is an eigen value of T if and only if det $[(T - \lambda I), B] = 0$, where $I : V \rightarrow V$ is the identity linear transformation and $dim\ (V) = n$.

(D) Attempt any one out of two : **5**

(i) Find eigen values and eigen vectors for the linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ defined by T (x, y, z) = $(-2y - 2z, -2x - 3y - 2z, 3x + 6y + 5z)$ by considering the standard basis of $\mathbb{R}^3$.

(ii) Let $T : V \to V$ be a linear transformation and let B be a basis of V. Then show that T is singular if and only if $det[T, B] = 0$.

5 (A) Answer the following questions in short. **4**

(i) The radius of curvature for y = log sec$\left(\frac{x}{c}\right)$ is ______.

(ii) The number of asymptotes of a curve of n^{th} degree is ______.

(a) equal to n (b) at least n

(c) at most n (d) None of these

(iii) The radius of curvature of the origin, if y-axis is the tangent at the origin, is given by ______.

(iv) A closed curve has-

(a) no asymptote

(b) one asymptote

(c) infinitely many asymptotes

(d) None of these

(B) Attempt any one out of two : **2**

(i) Find the multiple points of the curve

$x^3 + y^3 - 3xy + 3x + 3y = 1$.

(ii) Find the radius of curvature in Cartesian coordinate system.

(C) Attempt any one out of two : **3**

(i) Examine the Folium $x^3 + y^3 - 3axy = 0$ asymptotes.

(ii) Show that the curvature of a circle is constant.

(D) Attempt any one out of two : **5**

(i) Determine the asymptote of the general rational algebraic equation

$U_n + U_{n-1} + ... + U_1 + U_0 = 0$

where, U_r is a homogeneous expression of degree r in x, y.

(ii) Determine the condition for any point $(x, y)q$ to be a multiple point of the curve $f(x, y) = 0$.

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